

Residual Analysis for Comparing Procedures that Adjust Heterogeneous Phenotypic Variances in a 305 Days Lactational Model Genetic Evaluation

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1. Introduction

Proper modelling of heterogeneity of variance among strata of data and plausibility of its application in national evaluations act in opposite directions. Assuming a constant heritability and repeatability in a 305 day lactational model reduces computational needs because of a lower number of parameters to be estimated. This is the case with procedures that have been applied to national evaluations using a large amount of data as in Meuwissen *et al.* (1996) and Robert-Granié *et al.* (1999). These two procedures are preferred against those that perform a heterogeneity correction through standardization of data previous to genetic evaluation (Brotherstone and Hill, 1986; Weigel and Gianola, 1992) because they allow to take into account genetic and environmental effects that influence observations when estimating heterogeneity correction factors.

The procedure proposed by Meuwissen *et al.* (1996) has some drawbacks because its multiplicative model scales both fixed and random effects in the model. Robert-Granié (1999) states that the validity of this multiplicative model and particularly the quality of fit was still unknown. Pool and Meuwissen (2000) argue that scaling fixed effects could be quite problematic.

The objective of this study was to compare procedures for heterogeneity of variance correction under constant heritability and repeatability for its application in the Spanish national evaluation for production traits. Three alternative procedures were studied, the one proposed by Meuwissen *et al.* (1996), the modification proposed by Pool and Meuwissen (2000) and applied by Gengler *et al.* (2004) and the procedure proposed by Robert-Granié *et al.* (1999).

2. Materials and Methods

2.1. Data

Data used in this analysis correspond to those included in the national genetic evaluation of January 2001 for Holstein-Friesian cows in Spain. This data set includes records from first five lactations of cows calving from 1986 to late 2000. Completed lactations were required to have at least 215 days in milk and lactations in progress were extended and used in the evaluation only when production for at least 65 days and two tests were available. All lactations, complete or in progress, were projected to 305 days following Rekaya *et al.* (1996) and Wilmink and Outweltjes (1992). Total number of lactations were 1,375,759 and number of cows with data were 602,058. Number of animals in pedigree were 804,700. Kg of milk was the trait analyzed.

2.2. Models

Four different models were applied to the data. First of them was the model that assumes homogeneity of variance (HOM):

$$y = Xb + ZQg + Zu + Wp + e \quad (1)$$

where **b** is a vector of fixed effects, **g** the vector of genetic groups, **u** is the vector of additive genetic merit, **p** the vector of permanent environmental effects, **e** the vector of residuals and **X**, **Z** and **W** design matrices for fixed effects, additive genetic merit and permanent environment effects, respectively. **Q** is the matrix that associates animals with genetic groups to which parents belong in case they are unknown. $\text{Var}(\mathbf{u}) = \mathbf{A}\sigma_u^2$, where **A** is the additive genetic relationship matrix. and σ_u^2 is the additive genetic variance,

$\text{Var}(\mathbf{p}) = \mathbf{I}\sigma_p^2$, where σ_p^2 is the permanent effect variance and $\text{Var}(\mathbf{e}) = \mathbf{R} = \mathbf{D}\sigma_e^2$ where σ_e^2 is residual variance and \mathbf{D} a diagonal matrix that defines residual variances based on days in milk and calculated as square correlations between predicted and observed 305 days in the extension procedure.

The second model is that proposed by Meuwissen *et al.* (1996) (MEU1), where all effects are scaled, including fixed effects,

$$\mathbf{y}_i = (\mathbf{X}_i \mathbf{b} + \mathbf{Z}_i \mathbf{Qg} + \mathbf{Z}_i \mathbf{u} + \mathbf{W}_i \mathbf{p} + \mathbf{e}_i) \lambda_i \quad (2)$$

where i is a strata with observations with equal variance, \mathbf{e}_i is the residual random vector in strata i and \mathbf{X}_i , \mathbf{Z}_i and \mathbf{W}_i are design matrices for fixed effects, additive genetic merit, permanent environment effects in each strata, respectively. $\text{Var}(\mathbf{u})$, $\text{var}(\mathbf{p})$ and $\text{var}(\mathbf{e})$ are defined as in HOM.

In (2), λ_i is a scaling factor that affects dispersion of \mathbf{u} , \mathbf{p} and \mathbf{e} . Assuming constant heritability and repeatability in all strata and defining $\gamma_i = \log \lambda_i^2$, we can set the following linear model:

$$\gamma_i = \mathbf{S}_i \boldsymbol{\beta}$$

where $\boldsymbol{\beta}$ is a vector of unknown effects that can be fixed or random and \mathbf{S}_i is a design row vector.

The third model (MEU2) is an ‘ad hoc’ modification of the estimation algorithm of previous MEU1 model. This modification was suggested by Pool and Meuwissen (2000) and applied by Gengler *et al.* (2004). In each iteration, observations are pre-corrected with fixed effects obtained in the previous cycle, standardization is performed, fixed effects estimates are summed again and then the procedure carries on as in MEU1. Even though the estimation algorithm has not been derived by a probabilistic model, estimates correspond to a similar model than MEU1 where fixed environmental effects are not scaled. The general model would be:

$$\mathbf{y}_i = \mathbf{X}_i \mathbf{b} + (\mathbf{Z}_i \mathbf{Qg} + \mathbf{Z}_i \mathbf{u} + \mathbf{W}_i \mathbf{p} + \mathbf{e}_i) \lambda_i \quad (3)$$

defining a dispersion model for $\log \lambda_i^2$ similar to MEU1.

At last, the method proposed by Foulley (1997) and developed by Robert-Granié *et al.* (1999) was applied (ROBERT):

$$\mathbf{y}_i = \mathbf{X}\mathbf{b} + t_1 \sigma_{ei} \mathbf{Z}_i \mathbf{Qg} + t_1 \sigma_{ei} \mathbf{Z}_i \mathbf{u}^* + t_2 \sigma_{ei} \mathbf{W}_i \mathbf{p}^* + \mathbf{e}_i \quad (4)$$

where \mathbf{y}_i represents the observation vector in strata i , \mathbf{u}^* is a vector of standardized additive genetic merits, with $\text{var}(\mathbf{u}^*) = \mathbf{A}$, \mathbf{p}^* is the vector of standardized permanent environmental effects, with $\text{var}(\mathbf{p}^*) = \mathbf{I}_q$, and \mathbf{e}_i is the residuals vector in strata i , with $\text{var}(\mathbf{e}_i) = \mathbf{D}\sigma_{ei}^2$. Constants t_1 and t_2 are defined as:

$$t_1^2 = \frac{\sigma_u^2}{\sigma_e^2} = \frac{h^2}{(1-r)}$$

$$t_2^2 = \frac{\sigma_p^2}{\sigma_e^2} = \frac{r-h^2}{(1-r)}$$

$$\text{Now, } \log \sigma_{ei}^2 = \gamma_i = \mathbf{S}_i \boldsymbol{\beta}$$

When the three models were applied to the data, \mathbf{b} included the following factors: comparison group (herd-year-imported-parity-season), calving month and age at calving, these last two effects nested within parity and production system, defined as the combination of zones, periods and production levels. In $\boldsymbol{\beta}$ were included the fixed effects of production system-region, parity-age at calving, calving season, comparison group size and days in milk nested within complete or in progress status (DIM-CST), and the random effects of herd-year of calving (with a first order autoregressive structure defined by correlation ρ and variance σ_{hy}^2) and comparison group (with variance σ_{cg}^2). Average number of observation per herd-year and comparison group were 21 and 11, respectively.

In all procedures, heritability used in the analysis was 0.28 and repeatability 0.50 (Pena *et al.*, 2000).

In the three procedures applied for adjusting heterogeneity of variance, solutions were scaled to the estimated average factor for heterogeneity.

2.3. Estimation procedure

For the three adjustment procedures described before solutions for both, the unknowns in the mean model (\mathbf{b} , \mathbf{g} , \mathbf{u} , \mathbf{p}) and the unknowns of the dispersion model (β , σ_{hy}^2 , σ_{cg}^2 and ρ), had to be obtained.

2.3.1. Mean model equations

For MEU1 and MEU2, the usual mixed model equations (MME) assuming known heritability and repeatability have to be solved, but the RHS was obtained with observations adjusted by the heterogeneity factors $y_c = y e^{-0.5 \gamma_i}$. When applying MEU2, observations are corrected with fixed effects estimated in previous cycle of iteration, before standardization and then fixed effects estimates are sum again.

For ROBERT, the MME system was built as in Robert-Granié *et al.* (1999), where information from each strata is weighed by the corresponding error variance. This implies that both, the LHS and the RHS of the MME change in each cycle, while for MEU1 and MEU2 only the RHS changes.

2.3.2. Dispersion model equations

Following Meuwissen *et al.* (1996) and Robert-Granié *et al.* (1999), in each cycle of iteration, heterogeneity factors are estimated solving the following system of equations:

$$[\mathbf{S}' \hat{\mathbf{\Omega}}^{[t]} \mathbf{S} + \mathbf{\Lambda}] \beta^{[t+1]} = \mathbf{S}' \hat{\mathbf{\Omega}}^{[t]} \mathbf{s} \quad (5)$$

where $\mathbf{\Lambda}$ is a matrix including variances and first order autorregressive covariances for the herd-year effect in β and the variances for the comparison group effect:

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{H}^{-1} \sigma_{hy}^{-2} & 0 \\ 0 & 0 & \mathbf{I} \sigma_{cg}^{-2} \end{bmatrix}$$

where matrix \mathbf{H} includes correlations between years within a herd and it has a block diagonal structure \mathbf{H}_i , one for each herd, because random effects from different herds are considered independent.

Moreover, in (5), \mathbf{s} is a vector of “pseudo-observations” in each strata, that is obtained summing the last estimate of the heterogeneity factor obtained for the strata to the remaining heterogeneity in that strata:

$$\mathbf{s} = \mathbf{S} \beta^{[t]} + (\hat{\mathbf{\Omega}}^{[t]})^{-1} \mathbf{z}$$

$$\hat{\mathbf{\Omega}}^{[t]} = \text{diag}(\text{Var}(\mathbf{z}_i))^{[t]}$$

and calculating elements \mathbf{z}_i and $\text{Var}(\mathbf{z}_i)$ in each strata i in different way for MEU1, MEU2 and ROBERT, as described in Meuwissen *et al.* (1996) and Robert-Granié *et al.* (1999).

Solving (5) requires knowing the value of σ_{hy}^2 , σ_{cg}^2 and ρ . A REML scheme with iterative weights was applied on pseudo-observations, following the initial proposal of Engel *et al.* (1999) to obtain values for those unknowns at each cycle of iteration.

2.4. Solving strategy

A cycle of iteration while solving the system of equations for the mean and dispersion parameters consisted of one iteration on data (IOD) scheme used for the mean model equations, solving equations for the dispersion model by a direct method, using estimates of ρ , σ_{hy}^2 y σ_{cg}^2 from previous cycle, and one iteration for the REML scheme on pseudo-observations to update values for these parameters to be used in the subsequent cycle.

For MEU1, estimates calculated with MEU2 for ρ , σ_{hy}^2 and σ_{cg}^2 were be used, as preliminary simulation studies had shown that MEU1 would not obtain estimates less than one for ρ unless huge herd-year sizes are available.

2.5. Residuals Analysis

A fixed effects linear model was fitted to estimated residuals to search for possible sources of bias in the estimation of the unknowns of the mean model.

The efficiency of the procedures applied in correcting heterogeneity of variances was

checked using the logarithm of the cuasi-variance of estimated residuals within strata. Strata were defined by the combination of production system, region, parity number and length of lactation, grouping all lactations in progress in the same class to avoid too small strata. For each strata, a variable κ_i was defined as,

$$\kappa_i = \log \left(\frac{\sum_{j=1}^{n_i} \omega_j (\hat{e}_{ij} - \bar{e}_i)^2}{n_i - 1} \right)$$

where \hat{e}_{ij} is the estimated residual for observation j in strata i , \bar{e}_i is the average of estimated residuals in strata i , ω_j is a weight to consider differences in residual variances due to differences in days in milk and n_i is the number of observations in strata i .

Then, a fixed linear model was fitted to the calculated κ_i values, including production system, region, parity number and length of lactation effects. A minimum of 100 observations in each strata was required and number of observations in each strata were used as weights.

In both analyses, only estimated residuals from cows with both parent known were considered and effects were tested with F test and R^2 coefficients were obtained. All analyses were run with GLM procedure from SAS/STAT (1999).

2.6. Model comparison

Goodness of fit was measured through three criteria for general model comparison. First an estimate of mean square error was calculated as:

$$MSE = \left(\frac{\sum_{j=1}^n \omega_j \hat{e}_j^2}{\sum_{j=1}^n \omega_j} \right)$$

being now \hat{e}_j the estimated residual for observation j and n total number of observations. Secondly, an estimate of percentage of square bias (Ali y Schaeffer, 1987):

$$PSB = \frac{\sum_{j=1}^n \omega_j \hat{e}_j^2}{\sum_{j=1}^n \omega_j y_j^2}$$

And, thirdly, the correlation between observed and predicted values was calculated.

3. Results

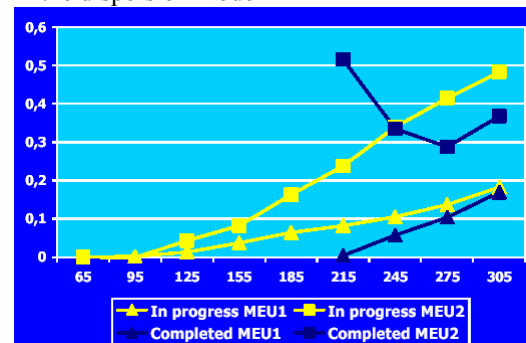
3.1. Parameters for the dispersion model

For MEU2, estimated values for ρ , σ_{hy2} and σ_{eg2} were, respectively, 0.85, 0.124 and 0.02. For ROBERT convergence was not achieved, even if fixing parameters estimated with MEU2. Therefore, this procedure was not considered in following analysis. However, it should be noted that ROBERT converged without problems when DIM-CST was not fitted in the dispersion model, even when parameters were not fixed.

3.2. Solutions for the dispersion model

Heterogeneity of phenotypic variance was found across classes of all effects considered in the dispersion model, with very similar magnitudes for MEU1 and MEU2, except for DIM-CST. Region-production system, management group size and lactation length were the factors with a larger impact on the heterogeneity of variances. Estimated differences in variability between classes of DIM-CST, for complete and in progress lactations with MEU1 and MEU2 are presented in Figure 1.

Figure 1. Solutions to days in milk effect nested within completed or in progress status (DIM-CST) in the dispersion model



For lactations in progress, variability increases with DIM for both procedures, but, changes with increasing DIM are much larger with MEU2.

For complete lactations (all of them projected to 305 days), MEU1 estimated differences in variability that also increased with length of lactation. MEU2 provided a different pattern in this case. A larger variability was associated for production when lactations had been completed at 215 DIM (minimum DIM for complete lactations) than for production in lactations completed at 305 DIM. The results obtained by MEU1 agree with the results reported by Urioste *et al.* (2001) and Guo *et al.* (2002). These authors found an inverse association between DIM and phenotypic variability of production and pointed to some problem in the extension procedure which would introduce some “extra-residual variation”. Larger variability associated with short complete lactations (215 DIM) could be due to a higher frequency of abnormal (low) records in the last available test, associated with the reason of disposal. This fact, linked with the large emphasis that the current extension procedure puts in these last tests, could result in an underestimation of 305 days production for many 215 DIM lactations. Consequently, short complete lactations are projected to 305 d adding a small amount or a large quantity depending mainly on the last recorded test, which may be abnormally low if the lactation was terminated due to the fact that the cow was culled, with a likely low level of production. An extra variability of 305 d production would be artificially generated in this case. This circumstance was not present in the data used by Van Raden *et al.* (1991) because they used only lactations with at least 305 DIM.

3.3. Residual Analysis

Modelling estimated residuals under HOM, MEU1 and MEU2 with a linear model including production system, region, lactation number, birth year and lactation length yielded R^2 coefficients of 3.4%, 1% and 3.1%, for the three procedures, respectively. This would indicate that the effects considered did not greatly affect the estimated residuals. This was expected because these factors, except

lactation length, were already considered in the mean model. Relevant discrepancies among procedures were found for factor DIM-CST. Least square solutions for the DIM-CST levels are presented in Table 1.

Table 1. Solutions for days in milk within complete status and F tests when modelling estimated residuals with HOM, MEU1 and MEU2 (*C=Completed; P=In Progress*).

DIM-CST	Method					
	HOM		MEU1		MEU2	
	Est	Pr>F	Est	Pr>F	Est.	Pr>F
P-65	-197,3	<,0001	-5,3	0,3316	-198,6	<,0001
P-95	-194,8	<,0001	-6,9	0,2092	-195,4	<,0001
P-125	-165,6	<,0001	-0,4	0,9269	-173,8	<,0001
P-155	-138,4	<,0001	-5,1	0,3246	-134,4	<,0001
P-185	-94,1	<,0001	-4,4	0,3908	-93,1	<,0001
P-215	-65,0	<,0001	-5,4	0,2854	-66,0	<,0001
P-245	-20,3	<,0001	-2,6	0,6050	-33,6	<,0001
P-275	-17,4	0,0025	-11,0	0,0521	-36,6	<,0001
P-305	49,4	<,0001	-1,4	0,7586	8,6	0,0646
C-215	-413,1	<,0001	-48,6	<,0001	-383,3	<,0001
C-245	-265,6	<,0001	-22,4	<,0001	-268,6	<,0001
C-275	-163,6	<,0001	-14,0	<,0001	-159,3	<,0001
C-305	0,0	,	0,0	,	0,0	,

The F tests provided are not exact because observations are not independent and error variances are not homogeneous, but, results can provide information of the magnitude of the bias associated with classes of DIM-CST with the three procedures applied. The solutions obtained under MEU1 did not differ significantly from zero, while HOM and MEU2 provided quite large values for the means of the lactation length classes, that significantly differed from zero. Production in 305 d for lactations completed with 215 DIM was around 400 kg milk lower than production for lactations completed with 305 DIM when HOM or MEU2 procedures were used while for MEU1 the maximum underestimation was 50 kg. Compared with complete lactations, estimated differences in 305 d production between lactations in progress were of much smaller magnitude for HOM and MEU2, which is logical due to the fact that frequency of last abnormal tests will be lower in a non completed lactation.

Consequences of the described bias on bull proofs were ascertained by computing DYD within DIM classes for the three procedures for

bulls with daughters in at least 10 herds . These DYD were modelled with a fixed linear model that included bull and DIM effect. DYD were weighed by number of daughters in each class. Results followed the same trend as in the previous analysis on residuals but estimated differences are of bigger magnitude (near - 1000 kg for complete lactations at 215 DIM under HOM and MEU2).

From the results in the residual and DYD analyses, the bias apparently present in the predicted lactation yield for short complete lactations under HOM and MEU2 was greatly reduced under MEU1. This might be explained by the fact that fixed effects are also scaled by the adjustment factors under MEU1. That implies that these factors adjust not only differences in variances between levels of effects in the dispersion model but also differences in means, even if the effect is not fitted in the mean model. This is due to the fact that solutions for fixed effects in the model on the mean are different for each observation, depending on the heterogeneity of variance stratum where the observation is recorded.

Fitting a linear model similar to the one fitted on residuals on the logarithm of the estimated variance of residuals to test the efficiency of the alternative adjustment procedures in reducing heterogeneity of variances resulted in R^2 coefficients of 94%, 51% and 34% for HOM, MEU1 and MEU2, respectively. These results show that both MEU1 and MEU2 correct a great part of the original heterogeneity of variance, but MEU2 is more efficient in this task. As for the residual analysis, solutions under the two adjustment procedures were similar for all factors except for DIM-CST. Table 2 shows the least square means and F tests for effects in the log-linear model on variance of residuals for the effect DIM-CST. It was observed that DIM-CST had a significant effect on the log variance under HOM or MEU1, while for MEU2, DIM-CST was not significant. Therefore, correction of heterogeneity of variance between DIM-CST classes was more efficient for MEU2.

Table 2. Solutions for days in milk effect (DIM) and F tests when modelling logarithm of quasi-variance of estimated residuals (*C=Completed*).

DIM	Method					
	HOM		MEU1		MEU2	
	Est.	Pr>F	Est.	Pr>F	Est.	Pr>F
Progress	-0,167	<.0001	-0,041	<.0001	0,017	0,0362
C- 215	0,096	<.0001	0,240	<.0001	-0,022	0,0437
C-245	-0,019	0,0327	0,104	<.0001	0,049	<.0001
C-275	-0,044	<.0001	0,043	<.0001	0,070	<.0001
C-305	0,000	.	0,000	.	0,000	.

3.4. Criteria for model comparison

Three criteria for general model comparison are shown in table 3. Although differences between these criteria were small, all three of them indicate that MEU1 provides a better fit, while differences between HOM and MEU2 were even smaller and favourable to MEU2. Ibáñez *et al.* (1999) also obtained small differences when a method for standardization of phenotypic variance previous to genetic evaluation was used against no standardization. In general, small differences between these criteria are expected when procedures differ basically in the dispersion parameters.

Table 3. Estimates for mean squared error (MSE), percentage of squared bias (PSB) and correlation between observed and predicted values for HOM, MEU1 and MEU2.

	HOM	MEU1	MEU2
MSE	469,665	440,724	460,311
PSB	$7.2 \cdot 10^{-3}$	$6.8 \cdot 10^{-3}$	$7.1 \cdot 10^{-3}$
r(y,y)	0.943	0.947	0.944

4. Conclusions

Results shown in this paper illustrate some features of the multiplicative mixed model. For these data and mean model, MEU1 provides a better fit, due to the multiplicative adjustment for classes of DIM-CST, what apparently eliminates the bias related to this factor.

MEU2 is more efficient in correcting heterogeneity of variance but it does not correct this bias. Because of this, and even taking into account, that with MEU1 parameters ρ and σ_{ra}^2 could not be estimated and MEU2 estimates had to be used for these parameters, MEU1 seems to be the method of choice for this data and mean model.

Implementation of a TDM in Spanish national production evaluation in 2006 will mitigate bias associated with DIM-CST, but this results are of interest if proposals of combining TDM and 305 model are followed (Guo *et al.*, 2002; Mrode *et al.*, 2004). Also, a procedure for detection of outliers in test day data should be applied.

5. References

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