Multiple Generation Selection for Nonlinear Profit Functions

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Abstract

Methods to obtain linear selection indexes for nonlinear profit functions that maximize cumulative net present value of profit over a planning horizon were described. Optimum indexes can are derived conform selection index theory but with economic values that are equal to a weighted average of partial derivatives of the profit function at trait means in future generations. Economic values can be derived using numerical procedures. Results were illustrated with an example.

Introduction

The traditional approach for development of multiple trait criteria to select for an overall economic objective is to derive a linear selection index (I) based on a linear breeding goal ($T=a'u$ where a is a vector of economic values and u is a vector of breeding values for traits of economic importance) as (Hazel, 1943): $I=b'X$ with $b=P^{-1}Ga$, where b is a vector of index weights, X is a vector with sources of information, $P=Var(X)$ and $G=Cov(X,u)$. The economic value of a trait in T is defined as the marginal effect on the objective (e.g. profit) of a marginal change in the population mean for a trait, while keeping all other traits in T constant. Economic values are generally derived as partial derivatives of the profit function, evaluated at current population means.

Moav and Hill (1966) showed that indexes derived based on the above principles did not maximize improvements in profit when profit was a nonlinear function of genetic traits and formulated the problem of derivation of a linear selection index that maximizes average profit in the next generation for nonlinear profit functions. Optimum indexes were shown to depend on achieved genetic gains, which in turn depend on index weights. Analytical derivation of optimum indexes was, therefore, not possible, but graphical methods were developed for selection on two traits (Moav and Hill, 1996). They also showed that similar procedures could be used when the objective was to maximize profit in the last generation of a planning horizon. Procedures were further formalized by Goddard (1983) and numerical solution procedures were developed by Itoh and Yamada (1988) and by Pasternak and Weller (1993).

Dekkers *et al.* (1995) showed that indexes of Moav and Hill (1966) can be derived using regular selection index procedures (Hazel, 1943) based on a linear breeding goal in which economic values are equal to partial derivatives of the profit function at trait means in the generation for which profit is maximized. Thus, if the objective is to maximize profit in the next generation, economic values are equal to partial derivatives at trait means in the progeny generation, rather than at trait means in the current generation. Numerical procedures similar to Pasternak and Weller (1993) can be used to derive economic values for such objectives.

Procedures discussed above can be used to derive selection indexes that maximize profit in the next generation or in a specific future generation. However, economic objectives of genetic improvement programs must consider both short and long term responses. Such an objective can be described as maximization of cumulative net present value of profit (CNPV) over a planning horizon. With CNPV, average profits in each future generation are discounted to present and summed. Because CNPV depends on genetic gains achieved in all future generations, maximization of CNPV involves simultaneous optimization of selection indexes for all generations in the planning horizon.

Linear indexes that maximize CNPV were derived by Dekkers *et al.* (1995) by formulating selection for nonlinear profit functions over multiple generations as an optimal control problem (Bryson and Ho, 1975). Resulting optimum indexes were found to be conform selection index theory for linear profit functions. The only distinction is in derivation of economic values.

Objectives of this paper are to summarize the main results of Dekkers *et al.* (1995) on selection indexes that optimize selection over multiple generations with nonlinear profit functions, to show the connection of derivation of such indexes with selection index theory, and to illustrate results.

Theory

Let CNPV over a planning horizon of T generations be represented by: $\pi = \sum_{t=1}^{T} w_t f(\mathbf{m}_t)$

where $f(\mathbf{m}_t)$ is average profit in generation t as a nonlinear function of $\mathbf{m}_t$, $\mathbf{m}_t$ is a qx1 vector of population means of q economic traits in generation t, and w_t is a discount factor [$=(1+\text{interest rate})^{-t}$].

Let I_t be the linear index for selection of parents in generation t: $I_t = \mathbf{b}_t' \mathbf{X}$, where, $\mathbf{X}$ is an nx1 vector of information sources, and $\mathbf{b}_t$ is an nx1 vector of index weights. Then the problem of finding index weights $\mathbf{b}_t$ that maximize π can be formulated in terms of an optimal control problem (see Dekkers *et al.* 1995):

$$\max_{\mathbf{b}_t, \mathbf{m}_t} \left\{ \sum_{t=0}^{T-1} [\, w_t f(\mathbf{m}_t)] + w_T f(\mathbf{m}_T) \right\} \qquad [1]$$

Subject to:

$$\mathbf{m}_{t+1} = \mathbf{m}_t + iG' \mathbf{b}_t/c \qquad \text{for } t = 0, ..., T\text{-}1 \qquad [1a]$$

$$\mathbf{b}_t' \mathbf{P} \mathbf{b}_t = c^2 \qquad \text{for } t = 0, ..., T\text{-}1 \qquad [1b]$$

Given $\mathbf{m}_0$

In [1], G is the matrix of covariances between X and economic traits in $\mathbf{m}_t$, P is the variance-covariance matrix of X, i is the selection intensity, c is an arbitrary constant which sets the standard deviation of the selection index to a fixed value, and $f(\mathbf{m}_T)$ represents the (salvage) value in the last stage of the planning horizon. Constraints [1a] represent responses to selection on index $\mathbf{b}_t'\mathbf{X}$ in each generation. Constraints [1b] force vectors $\mathbf{b}_t$ to a unique solution. Choice of c will result only in a proportional scaling of $\mathbf{b}_t$.

Solutions for $\mathbf{b}_t$ (for t=0,...,T-1) that maximize [1] satisfy (see Dekkers *et al.* (1995)):

$$\mathbf{b}_t = k\, \mathbf{P}^{-1}\mathbf{G}\, \mathbf{a}_t \qquad [2]$$

where k is a scaling factor and the vector of economic weights is equal to:

$$\mathbf{a}_t = \sum_{v=t+1}^{T} w_v \delta f(\mathbf{m}_v)/\delta \mathbf{m}_v \qquad [3]$$

Equation [2] is identical to the usual selection index equations (Hazel, 1943), which shows the derivation of selection indexes for nonlinear profit functions differs from derivation of selection indexes for linear profit functions only with regard to computation of economic values (equation [3]).

When the objective is to maximize CNVP, economic values are proportional to the weighted average of partial deriviatives of the profit function at population trait means in future generations. The weight on the partial derivative in generation v is equal to the weight on average profit from generation v in the overall objective function. For CNVP, weights are equal to the discount factors.

Economic values in equation [3] depend on future population means, which depend on genetic improvement in previous generations. Economic values and optimum index weights can, therefore, not be derived analytically. A numerical procedure was provided by Dekkers *et al.* (1995).

Equations [2] and [3] apply to any number of traits in the breeding goal and index and when traits in the index differ from traits in the breeding goal. Procedures also apply to any objective function that can be formulated as a linear function of average profit in generation t ($t=1,\ldots,T$). Results simplify when the objective is to maximize profit in a given generation. For example, when the objective is to maximize profit in the next generation, $T=1$ and $w_1=1$, and, based on equation [3], economic values are equal to partial derivatives of the profit function at population means in the next generation. Similarly, when the objective is to maximize profit in generation T, $w_t=0$ for $t=1,\ldots,T-1$, $w_T=1$, and economic values are equal to partial derivatives of the profit function at population means in generation T. In this case, a constant index results for the planning horizon, which confirms the result of Moav and Hill (1966). In all cases, once economic values have been obtained, optimum index weights can be derived using the regular selection index equations (equation [2]).

Example

The impact of selection for a nonlinear profit function over multiple generations will be illustrated here based on the example used by Dekkers *et al.* (1995). Selection was for rate of lay (RL) (%) and egg weight (EW) (g/egg) in poultry, with the following function for mean profit in generation t:

$$f(RL_t, EW_t) = 3.11\, RL_t\, EW_t\, (p_{EWt} - c_f) - c_m \quad (\$/\text{bird/year})$$

where, RL_t and EW_t are population mean RL and EW in generation t, 3.11 is the number of eggs per year per percent RL, p_{EWt} is mean return per gram of egg, c_f is variable feed cost (= \$.0008621/g egg), and c_m is the maintenance cost per bird per year (ignored in the current study). A logistic function was derived for p_{EWt} based on the categorical pricing scheme for eggs in Canada (Figure 1):

$$p_{EWt} = .0821\, e^{-11.997+.258\,EW_t}/[(1+e^{-11.997+.258\,EW_t})EW_t],$$

Phenotypic variances for RL and EW of 40.88 and 18.42, heritabilities of 0.18 and 0.74, and phenotypic and genetic correlations of -0.17 and -0.29 were used. Mass selection over 10 generations with a selection intensity of 1 was evaluated. A discount rate of 5% per generation was used.

Table 1 summarizes results for the selection strategy that maximized CNPV. The table illustrates that, for the optimum selection strategy, economic values are a weighted average of partial derivatives at population means in future generations (economic values in Table 1 are based on equation [3], divided by the sum of weights). For example, the economic value of egg weight in generation 0 (0.311) is a weighted average of partial derivatives at mean egg weight in generations 1 through 10. Economic values for the strategy that maximizes CNPV take into account that partial derivatives of the profit function decrease at future trait means, which reduces the economic value of improving egg weight in generation 0 relative to the partial derivative in generation 0 (1.187).

Table 2 compares results four strategies to derive economic values:
A) economic value = partial derivative at trait mean in the current generation,
B) economic value = partial derivative at trait mean in the next generation,
C) economic value = partial derivative at trait mean at the end of the planning horizon,
D) economic value = weighted average of partial derivatives at trait means in future generations.

Whereas strategy A reflects what is most often used in practise, strategies B, C, and D correspond to three different objective functions: maximization of profit in the next generation (B), maximization of profit in the last generation (C), and maximization of CNPV (D).

The four selection strategies resulted in large differences in economic values, in particular in early generations (Table 2). In generation 0 the economic value for egg weight was almost four times lower for strategy D than for A and B. In contrast, the economic value of rate of lay was almost three times greater for strategy D than for B. Differences between strategies diminished over generations. Strategy C resulted in constant economic values and index weights for all generations. The economic value of egg weight was low (0.032) because mean egg weight reached in generation 10 was close to the optimum. The economic value of rate of lay was greater for C than for other strategies.

Differences in economic values resulted in similar differences in index weights (Table 2). In generation 1, relative emphasis on egg weight was almost four times lower for strategy D than for B. However, differences in index weights resulted in only small to moderate differences in genetic gain.

Strategies A and B put high emphasis on EW in early generations and moved the population close to the optimum egg weight after four generations (Table 2). However, this was at the cost of rate of lay, which decreased by over 2% in four generations. Strategy A clearly broke down when the population was close to the optimum egg weight. Strategy D did not reach the optimum EW as quickly. But the 'cost' of high emphasis on EW with strategies A and B, in terms of reduced rate of lay in early generations, was less for strategy D. Strategy C considered only long term responses and genetic improvement was linear for both traits (Table 2).

Strategy D resulted in the greatest CNPV (Table 2), as expected, but differences with B were small (.3%). Strategy A also performed surprisingly well. Maximization of profit in generation 10 (C) resulted in over 20% lower CNPV than D, but had highest profit in generation 10.

Discussion

The theory illustrated above is based on objectives that are a function of trait means. Elsen *et al.* (1986), however, argued that the objective should be to maximize mean profit, rather that profit evaluated at trait means. Itoh and Yamada (1988) showed that these two objectives result in identical optimum indexes for linear and quadratic profit functions. For other profit functions results are not identical, although differences may be small (Itoh and Yamada, 1988).

The theory and example used in this paper applies to discrete generations. Principles can, however, be extended to overlapping generations. Recently, Gibson *et al.* (1995) used a gene-flow model of the Canadian Holstein population, coupled with a nonlinear optimization program, to derive economic values for milk, fat and protein yield in each year of a planning horizon of 20 years for alternative scenarios of evolving market demands for fat and protein. Situations in which milk, fat, and/or protein yield per cow increased over time due to improved management were modelled also. Problems were formulated as a nonlinear programming problem conform equations [1].

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Table 1. Population means by generation, partial derivatives of the profit function at those means, economic values, and relative index weights for a selection strategy for egg weight and rate of lay that maximizes cumulative net present value of profit over 10 generations.

Generation	Discount factor	Population mean		Egg weight		Rate of lay		Ratio of index weights (weight/rate)
		Egg weight	Rate of lay	Part. deriv.	Econ. value	Part. deriv.	Econ. value	
0	---	45.00	90.00	1.187	0.311	-.017	0.075	13.52
1	0.952	48.15	89.45	1.169	0.190	0.025	0.083	8.06
2	0.907	51.27	88.98	0.788	0.098	0.060	0.086	3.93
3	0.864	54.23	88.75	0.379	0.049	0.079	0.087	1.69
4	0.823	56.48	89.05	0.147	0.030	0.086	0.087	0.77
5	0.784	57.60	89.85	0.061	0.022	0.087	0.087	0.42
6	0.746	58.04	90.84	0.032	0.020	0.087	0.087	0.29
7	0.711	58.20	91.61	0.022	0.019	0.087	0.087	0.24
8	0.677	58.26	92.99	0.019	0.018	0.087	0.087	0.22
9	0.645	58.27	94.09	0.018	0.018	0.087	0.087	0.22
10	0.614	58.28	95.18	0.018	---	0.087	---	---

Table 2. Economic weights, responses to selection, and profit for 4 selection strategies

	Generation	A	B	C	D
Economic value egg weight	0	1.187	1.167	0.032	0.311
	3	0.346	0.080	0.032	0.049
	9	-0.071	0.018	0.032	0.018
Economic value rate of lay	0	-0.017	0.025	0.087	0.075
	3	0.080	0.087	0.087	0.087
	9	0.087	0.087	0.087	0.087
Ratio of index weights (b_{EW}/b_{RL})	0	88.67	50.36	.88	13.52
	3	14.03	3.12	.88	1.68
	9	-4.42	.22	.88	.22
Genetic level for egg weight (g/egg)	1	48.16	48.16	46.31	48.15
	4	57.63	57.31	50.23	56.48
	10	57.05	58.27	58.08	58.28
Genetic level for rate of lay (%)	1	89.34	89.35	90.73	89.45
	4	87.51	88.06	92.91	89.05
	10	90.13	94.41	97.29	95.18
Average profit ($/bird/year)	1	3.84	3.84	1.60	3.83
	4	9.19	9.22	6.22	9.21
	10	9.37	9.82	10.07	9.89
CNPVP ($)	10	52.13	53.51	40.68	53.71

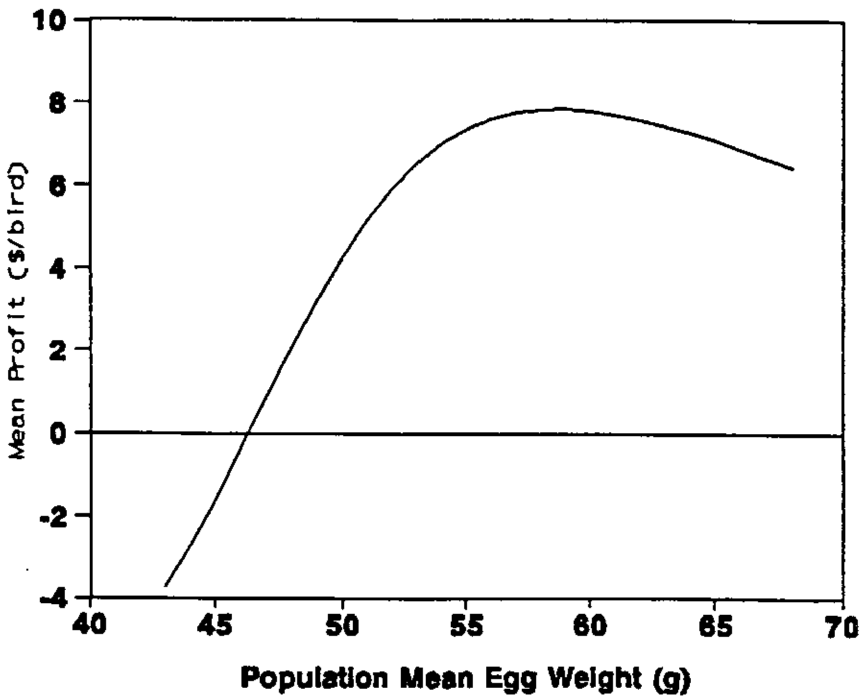

Figure 1. Effect of mean egg weight on mean profit at rate of lay of 90%.