Deriving Simple Equations for Evaluations of Bulls in the Interbull Mace System

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Introduction

Animal model evaluations of bull proofs produced in each participating country are passed to the Interbull Centre to be used for Multi Trait Across County Evaluations (MACE). Since animal model evaluations are done by each country, it is possible for each country to thoroughly investigate the proof of a bull in terms of how the various sources of information contributing to a bull=s proofs are weighted, when necessary. Van Raden and Wiggans (1991) presented derivations for the contribution of various sources of information to animal model evaluations.

However since MACE are carried out at the Interbull Centre, the calculation of Interbull proofs, especially for foreign bulls in importing countries, seems like a ›black box= since individual countries have little or no information about such bulls. For wider acceptance and use of MACE, it will be beneficial if the contributions of various sources of information to the proof of bulls can be explained. This paper presents simple equations indicating the contributions of various sources of information to the MACE proofs of bulls under different situations. It will concente on the proofs of foreign bulls in importing countries since Interbull and national proofs for home bulls are similar in each country.

Initially for simplicity, consider the equation for the proof of a bull under the Interbull multivariate analyses, assuming only 2 countries. Also assume the bull has no progeny.

$$\begin{bmatrix} r^{11}+ng^{11} & ng^{12} \\ ng^{21} & r^{22}+ng^{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = G^{-1} \begin{bmatrix} v_1(a_{s1}+v_2(a_{k1}+a_{j1})) \\ v_1(a_{s2}+v_2(a_{k2}+a_{j2})) \end{bmatrix} + \begin{bmatrix} r^{11}(y_1-c_1) \\ r^{22}(y_2-c_2) \end{bmatrix} \qquad (1)$$

where

r^{ii}	=	number of daughters in country i multiplied by the residual variance for i^{th} country
G^{-1}	=	inverse of the additive genetic covariance matrix
n	=	diagonal elements of A^{-1}
v_i	=	off-diagonal elements of A^{-1} between bull and parents with signs reversed
a_1,a_2	=	proofs for bulls of country 1 and 2 respectively
a_{si} , a_{ki} and a_{ji}	=	solutions for sire, maternal grandsire and maternal granddam in country i
y_i	=	deregressed proof in country i
c_i	=	fixed effect of country i

Situation A:

Bull has proof only in country 1 (foreign country). Proof in importing country (country 2) is:

$$ng^{22} a_2 = g^{22} (v_1 a_{s2} + v_2 (a_{k2} + a_{j2}) + g^{21} (v_1 (a_{s1}) + v_2(a_{k1} + a_{j1})) - ng^{12} a_1$$

$$g^{22} a_2 = g^{22} (2 \; a_{s2} + 3 \; (a_{k2} + a_{j2})) - g^{12} (a_1 - (2 \; a_{s1} + 3 \; (a_{k1} + a_{j1})))$$

$$= g^{22} (PA_2) - g^{12} (a_1 - PA_1)$$

with

$$PA_2 = 2 \; a_2 + 3 \; (a_{k2} + a_{j2}) \text{ and}$$

$$PA_1 = 2 \; a_1 + 3 \; (a_{k1} + a_{j1})$$

$$a_2 = PA_2 - g^{12}/g^{22} \; (a_1 - PA_1) \qquad (2)$$

Equation [2] can be generalised if bull has proof in n countries but no proof in j as

$a_j \; = \; PA_j \; - \; g^{ij}/g^{jj} \; (a_i \; - \; PA_i) \quad i = 1, \; n; \; i$ ¥ j

In the bivariate situation, weight on parent average in the importing country and Mendelian Sampling effect from foreign country is in the ratio of 1 : $g^{12}/g^{12.}$ For example, assuming a bivariate analysis between UK (importing country) and USA, this ratio will be 1 : 0.27 using the genetic parameters from the Interbull February 1997 run. Equation 2 is illustrated below by calculating the proofs of 4 USA bulls with proofs only in the USA but no proof in the UK.

Table 1. Interbull Protein* proofs for 4 bulls and their parents in UK and USA.

Bull	**Proof *** **kgs lbs**	**Sire**	**Proof *** **kgs lbs**	**Maternal grand sire**	**Proof *** **kgs lbs**
BARLO	29.7 (83.8)	LEADMAN	21.7 (44.1)	ROYALTY	17.6 (24.1)
DAVID	27.8 (52.9)	MASCOT	36.2 (64.2)	CLEITUS	24.3 (35.2)
CHECKMATE	31.9 (87.7)	OSDEC-ENDEAVOUR	26.9 (63.0)	BLACKSTAR	20.1 (30.0)
DIEGO	32.10 (71.8)	MASCOT	36.2 (64.2)	BLACKSTAR	20.1 (30.0)

* UK Interbull proofs in February 1997 for protein, USA Interbull proofs in brackets

Using equation 2, proofs for these bulls are:

BARLO = 2 (21.7) + 3 (17.6) - K (83.8 - (2 (44.1) + (24.1))) = 30.1

$K = g^{12}/g^{22} = -0.267$

DAVID = 2 (36.2) + 3 (24.3) - K (52.9 - (2 (64.2) + 3 (35.2))) = 27.4

CHECKMATE = 2 (28.9) + 3 (20.1) - K (87.7 - (2 (63.0) + 3 (30.0))) = 32.5

DIEGO = 2 (36.2) + 3 (20.1) - K (71.6 - (2 (64.2) + 3 (30.0))) = 31.7

Equation [2] indicates that parent average in the importing country could greatly influence the proof of young bulls in these countries. Considering the 4 bulls in Table 1, the ranking of these bulls differ greatly in the USA and UK. Although the proof of Barlo and David differ by 30.9 lbs of protein in the USA, Barlo is only 1.9 kg higher in the UK. Also while Checkmate is 16.1 lb of protein superior to Diego, it is -0.2 kg lower in the UK. These results are mainly due to the very high UK proof of Mascot, the sire of both David and Diego. This implies that if the high proof of Mascot in the UK is due to preferential treatment as a result of the high price of semen or for some other reason, the effect is expressed on the proofs of Mascot sons in the UK in the MACE system. In this situation, Interbull evaluations carry over the effect of any preferential treatment on secondary proofs of bulls in importing countries to their sons. Table 2 shows the simple and rank correlations between the UK and USA, Canada and Germany Interbull proofs for 2 categories of bulls with national proofs only in their respective foreign countries in the Interbull analysis.

a) Top 100 protein bulls in each of the three foreign counties which are sons of bulls (sires) with UK national proofs in the Interbull evaluation and

b) Top 100 protein bulls in each of the three foreign counties that are sons ofbulls (sires) without any national UK proof.

Table 2. Simple and rank correlations between Interbull proofs for two categories of bulls in UK.

	Sons of Sires with UK proof		**Sons of Sires without UK proof**	
	r^1	**r^2**	**r^1**	**r^2**
CANADA	0.76	0.69	0.96	0.93
USA	0.57	0.56	0.79	0.76
GERMANY	0.85	0.84	0.91	0.87

r^1 = simple correlation, r^2 = rank correlation

In general the lower rank correlations between the UK and the foreign proofs for the group of bulls which are sons of sires with a national UK proof in the Interbull evaluation tend to confirm the influence of parental proofs in UK on these young bulls. It may be necessary for Interbull to re-introduce a strategy to limit the influence of secondary proofs of imports on MACE of bulls.

Situation B:

Bull has proof in both foreign and importing countries. The proof of the bull in country 2 (importing country) following similar arguments as in situation A is:

$$(n^{-1} r^{22} + g^{22})\, a_2 = g^{22}(PA_2) - g^{12}(a_1 - PA_1) + n^{-1} r^{22}(y_2 - c_2)$$

$$(n^{-1} d_2/\sigma^2_{e2} + g^{22})\, a_2 = g^{22}(PA_2) - g^{12}(a_1 - PA_1) + n^{-1} d_2/\sigma^2_{e2}(y_2 - c_2)$$

where d_2 = number of daughters for the bull in country 2

Multiply both sides of equation by $16\,\sigma^2_{e2}$:

$$(t\,d_2 + 16\%_2)\ a_2 = 16\,\%_2\ (PA_2) - 16\ \%_3\ (PA_1) + t\,d_2\,(y_2 - c_2) \qquad (3)$$

where

$\%_2 = g^{22}\sigma^2_{e2}$, $\%_3 = g^{12}\sigma^2_{e2}$

t = 11 when sire and mgs are known, 15 when only mgs is known
12 when only sire is known and 16 when none are known

Thus the ratio of the weights on parent average from importing country, foreign country and proof in importing country is:

$$\frac{16\,\%_2}{D} \quad : \quad \frac{16\,\%_3}{D} \quad : \quad \frac{t\,d_2}{D}$$

where $D = td_2 + 16\,\%_2$

Note that equation (3) can be generalised if the bull has proofs in k countries as

$$a_j = D_j\,(16\,\%_j\ (PA_j) - 16\ \%_i\ (PA_i) + td_j\,(y_j - c_j)$$

with

$\%_i = g^{ij}\sigma^2_{e2j}$, $i = 1, k;\ i \neq j$

$\%_j = g^{jj}\sigma^2_{ej}$ and $D_j = 1/\ (td_j + 16\,\%_j)$

Using UK (home country) and USA as an example in a bivariate analysis with the genetic parameters for protein from the Interbull Feb. 1977 run of; $g_{11} = 457.10$, $g_{12} = 121.99$, $g_{22} = 39.31$ and σ^2_{e2} (calculated as $(4 - h^2_2/h^2_2)\ g_{22}$), the ratio of weights on PA_2 (parent average in importing country), $a_1 - PA_1$ (Mendelian sampling effects in foreign country), y_2 (deregressed proof from importing country) varies with changes in the number of daughters (see Table 3).

Table 3. Ratio of weights on different components of information under MACE when a bull has national proofs in both countries in a bivariate situation.

No of daughters	PA_2	$(a1\text{-}PA_1)$	y_2
20	0.82	0.22	0.18
60	0.60	0.16	0.40
100	0.47	0.13	0.53
300	0.23	0.06	0.77
600	0.13	0.03	0.87
1000	0.08	0.02	0.92
1520	0.05	0.01	0.95

The influence of parent average in importing country tends to be high when an imported bull has few daughters in his secondary proof. Thus preferential treatment on parents could still affect the proof of the bull in the importing country with few daughters. However as the number of daughters increases (> 300) the secondary proof dominates with little influence from the primary proof in country of origin through Mendelian sampling effects and also the parent average in importing country.

Situation C:

Bull has proof in both countries but in addition is a sire of sons in country 1 (foreign country) only. Proof of bull in country 2, following similar arguments in situations A and B is:

$$((r^{22} + w_1\ g^{22}P)\ n^{-1} + g^{22})\ a_2 = g^{22}\ (PA_2) - g^{21}\ (a_1 - PA_1) - (n^{-1}\ w_1 + 1)\ g^{12}\ a_1 + n^{-1}\ r^{22}\ (y_2 - c_2) + 3_p\ g^{21}\ (\ 2\ \ a_{01} - \ c\ \ (a_{m1}) + 3_p\ g^{22}\ (\ 2\ \ a_{02} - \ c\ \ (a_{m2}))$$

where

w_1 = element of A^{-1} = 4/11 when mates of bull (mgs) are known

a_{m1} = $(a_{mgs1} + a_{mgd1})$; a_{m2} = $(a_{mgs2} + a_{mgd2})$

a_{oi} = Interbull proof of o^{th} male progeny of bull in country i

P = number of male progeny

$$(n^{-1}\ r^{22} + 3\ \ g^{22}P + g^{22})\ a_2 = g^{22}\ (PA_2) - g^{21}\ (a_1 - PA_1 + 3\ \ a_1) + n^{-1}\ r^{22}\ (y_2 - c_2) + 3_p\ \ 2\ g^{21}\ \ (a_{01} - 3\ \ (a_{m1})) + 3_p\ \ 2\ g^{22}\ \ (a_{02} - 3\ \ (a_{m2}))$$

Multiply both sides of equation by $16\ \sigma^2 e_2$

$$(td_2 + 4\%_2\ (P{+}4))\ \ a_2 = 16\ \ \%_2\ (PA_2) - 16\ \%_3\ \ (a_1\ \ PA_1 + 3\ \ a_1) + td_2\ (y_2 - c_2) + \ 3_p 8\ \ \%_3\ (a_{01} - 3\ \ (a_{m1})) + 3_p 8\ \%_2\ \ (a_{02} - 3\ \ (a_{m2})) \quad (4)$$

The ratio of weights on parent average in importing country, Mendelian sampling for bull in foreign country plus component due to his progeny, the bulls proof in the importing country, progeny contributions for foreign and importing countries respectively are

$$\frac{16\ \%_2}{D} : \frac{16\ \%_3}{D} : \frac{t\ d_2}{D} : \frac{3_p 8\ \%_3}{D} : \frac{3_p 8\ \%_2}{D}$$

where $D = td_2 + \ 4\ \%_2\ (P + 4)$

Again using USA and UK as examples, the ratio of these weights with varying number of daughters in importing country and male progeny in foreign country are shown below.

Table 4. Ratio of weights on different components of information under MACE when a bull has national proofs in both countries and sons with proofs in a bivariate situation.

No of daughters in Country 2	No of Sons in Country 1	PA_2	$a_1 - PA_1 + 1/4\ a_1$	y_2	Progeny Contribution Foreign country	Importing country
20	1	0.68	0.18	0.15	0.09	0.34
	5	0.40	0.11	0.09	0.27	1.01
	10	0.27	0.07	0.06	0.36	1.34
100	1	0.42	0.11	0.48	0.06	0.21
	5	0.30	0.08	0.33	0.20	0.74
	10	0.22	0.06	0.24	0.29	1.08
500	1	0.14	0.03	0.82	0.02	0.07
	5	0.13	0.03	0.72	0.08	0.32
	10	0.11	0.03	0.62	0.15	0.55
1000	1	0.08	0.02	0.90	0.01	0.04
	5	0.07	0.02	0.83	0.05	0.18
	10	0.07	0.02	0.76	0.09	0.34

Firstly, note that equation 4 holds and therefore the ratio of weights in Table 4 regardless of whether the sons were born in the foreign or importing country. With just one son, the parentage in importing country will dominate with few daughters (20) but at 100 daughters the ratio of weights on PA_2 and y_2 are equal,. As the number of daughters increases ($\geq$500), the information from daughters dominates.

With 5 or more sons and few daughters ($\leq$100), the influence of information from the sons seems to dominate. However, with 1000 or more daughters, the influnce of information from sons drops sharply and daughters influence is far more important. At about 500 daughters and 10 sons, the ratio of weights on information from sons and daughters are about the same.

Conclusion

The various equations presented are helpful in trying to understand the contribution of several sources of information to the Interbull MACE proof of a bull. Unlike the univariate situation where the sum of the weights on sources of information add up to unity, this is not the case in multivariate analysis. However the ratio of weights do indicate the possible large influence of parent average in importing countries on the Interbull proof of foreign bulls with little or no information in the importing country. Interbull should consider limiting the influence of secondary proofs on the International proof of bulls by giving a lower weight to information from importing countries.

Reference

Van Raden, P. M. and Wiggans, G. R. 1991. Deviation, calculation and use of national animal model information. *J. Dairy Sci. 74,* 2737-2746.