# A Simple Approximation to the Reliability of Interbull Proofs for Foreign bulls

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In computing breeding values, account is taken of the reliability of different sources of information. In choosing bulls to use, farmers can ignore reliability, provided the breeding values are not biased from favoured treatment but, in practice, many farmers place considerable importance on reliability.

If we wish to merge Interbull proofs with more recent home country information, we need to know the reliability of the proof that includes the foreign information. If the Interbull reliability is too high, too much weight is given to the foreign information.

The inverse of the MACE equations provides exact estimates of the prediction error variance, and hence effective daughters and reliability. With large numbers of bulls and countries, this becomes computationally difficult, so approximations are required.

Concerns were raised that some bulls with no Australian progeny had reliabilities that seemed too high. To check if these concerns were valid, a simple approximation using selection index theory was developed.

## Selection index

The standard selection index equations provide one method of computing an approximate reliability.

Calculation of a selection index relies on the correlations between performance in different countries. The correlations used by Interbull between performance in different countries are given in Table 1.

Table 1. Correlations between breeding values in different countries.

| | CAN | DEU | DNK | FIN | FRA | ITA | NLD | SWE | USA | CHE | GBR | NZL | AUS | AUT | BEL | IRL | ESP | CSK | SLO |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CAN | 1.0 | .92 | .91 | .93 | .95 | .93 | .93 | .91 | .96 | .89 | .93 | .80 | .84 | .87 | .88 | .91 | .92 | .89 | .88 |
| DEU | | 1.0 | .91 | .90 | .92 | .90 | .93 | .90 | .89 | .90 | .90 | .77 | .80 | .91 | .89 | .88 | .90 | .91 | .89 |
| DNK | | | 1.0 | .94 | .91 | .90 | .94 | .94 | .91 | .90 | .93 | .76 | .81 | .89 | .91 | .92 | .91 | .90 | .88 |
| FIN | | | | 1.0 | .90 | .90 | .94 | .94 | .91 | .89 | .91 | .77 | .80 | .89 | .90 | .90 | .91 | .89 | .87 |
| FRA | | | | | 1.0 | .95 | .93 | .91 | .95 | .89 | .93 | .78 | .82 | .88 | .91 | .90 | .93 | .89 | .88 |
| ITA | | | | | | 1.0 | .91 | .90 | .95 | .90 | .91 | .77 | .81 | .87 | .88 | .89 | .93 | .88 | .88 |
| NLD | | | | | | | 1.0 | .94 | .93 | .92 | .93 | .80 | .83 | .89 | .92 | .92 | .90 | .90 | .87 |
| SWE | | | | | | | | 1.0 | .90 | .88 | .91 | .76 | .80 | .88 | .90 | .88 | .89 | .89 | .88 |
| USA | | | | | | | | | 1.0 | .91 | .91 | .77 | .81 | .87 | .88 | .89 | .91 | .90 | .88 |
| CHE | | | | | | | | | | 1.0 | .89 | .76 | .80 | .89 | .87 | .90 | .88 | .88 | .87 |
| GBR | | | | | | | | | | | 1.0 | .81 | .84 | .88 | .89 | .95 | .88 | .89 | .89 |
| NZL | | | | | | | | | | | | 1.0 | .90 | .79 | .77 | .82 | .78 | .78 | .77 |
| AUS | | | | | | | | | | | | | 1.0 | .81 | .82 | .83 | .82 | .80 | .79 |
| AUT | | | | | | | | | | | | | | 1.0 | .88 | .88 | .89 | .91 | .89 |
| BEL | | | | | | | | | | | | | | | 1.0 | .89 | .91 | .89 | .89 |
| IRL | | | | | | | | | | | | | | | | 1.0 | .89 | .89 | .88 |
| ESP | | | | | | | | | | | | | | | | | 1.0 | .88 | .88 |
| CSK | | | | | | | | | | | | | | | | | | 1.0 | .89 |
| SLO | | | | | | | | | | | | | | | | | | | 1.0 |

To simplify the selection index equations, the breeding values can be expressed in units of genetic standard deviation.

If we have breeding values in foreign countries, the breeding value in Australia can be predicted from a selection index

$$b = P^{-1}G$$

where

$P$ gives the correlation among the proofs available, and

$G$ gives the correlation between the foreign proof and the true genetic merit in Australia.

**Example 1**

Breeding values available in 2 countries for a bull with no Australian daughters.

Assuming that :

the breeding values are expressed in units of genetic standard deviation,

the reliabilities in the 2 countries are $R_1^2$ , $R_2^2$

the genetic correlations between performance in different countries are $r_{12}$ , $r_{1A}$ , $r_{2A}$

then $P$ and $G$ are :

$$P = \begin{bmatrix} R_1^2 & R_1^2 R_2^2 r_{12} \\ R_1^2 R_2^2 r_{12} & R_2^2 \end{bmatrix} G = \begin{bmatrix} R_1^2 r_{1A} \\ R_2^2 r_{2A} \end{bmatrix}$$

The reliability of the breeding value in Australia is given by

$$R_A^2 = R_{A1}^2 = b'Pb$$

For a bull with very reliable proofs (reliability of 100%) in New Zealand and Canada, the equations are:

$$b = \begin{bmatrix} 1 & .8 \\ .8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} .84 \\ .9 \end{bmatrix} = \begin{bmatrix} .333 \\ .633 \end{bmatrix}$$

and the reliability = 0.850.

**Example 2**

Breeding values available in 1 foreign country for a bull with no Australian daughters.

$$P = R_1^2 \quad and \quad G = R_1^2 R_{A1}^2$$

$$b = r_{A1}$$

and

$$R_A^2 = b'Pb = r_{1A}^2 R_1^2$$

In other words, the Australian reliability equals the foreign reliability times the square of the apparent genetic correlation between performance in Australia and in the other country.

For a bull with a reliable New Zealand proof, the reliability would be 0.9 times 0.9 (i.e. 0.81) times the New Zealand reliability.

**Example 3**

Breeding values available in several foreign countries for a bull with no Australian daughters.

If the bull has a very reliable proof in every other country $P$ would be an 18 by 18 matrix, $G$ an 18 element vector, $b$ = [0.49, -0.19, 0.19, -0.26, -0.15, 0.11, 0.04, -0.02, -0.23, 0.14, 0.18, 0.60, 0.09, 0.25, -0.23, 0.06, -0.00, -0.08]

More importantly the reliability would be 0.869. Given proofs in Canada and New Zealand, the partial correlation between the Australian breeding value and most other proofs is close to zero.

## Lower reliabilities

Where the reliabilities in each country are substantially less than 1, there is more to gain from adding information from several countries. For example, if the reliabilities of a bull's proofs in New Zealand, Canada, and USA are each 0.70, the reliability for the Australian proof would be 0.588, while one with information in only New Zealand and Canada would be 0.534.

## Inclusion of Australian relatives

Even if the bull had a sire and maternal-grand-sire with reliable Australian proofs, the reliability of the bull's breeding value in Australia would only be 0.877.

In this case, the sire is easily included by adding a row and column to the $P$ matrix, linking the proof in each country with the sire's (and/or maternal-grand-sire's) breeding value in Australia.

Only for bulls with Australian daughters, or several Australian sons with proofs, can the reliability be much greater than 0.88.

A summary of the effect of including different amounts of foreign information is given in Table 2.

Table 2. Effect of foreign information on Australian reliabilities.

| Countries with proof | Reliability in Australia with foreign reliability equal to 1.000 | Reliability in Australia with foreign reliability equal to 0.700 |
|---|---|---|
| NZL | 0.81 | 0.441 |
| **NZL, CAN** | 0.850 | 0.534 |
| **NZL, CAN, USA** | 0.850 | 0.588 |
| **All except AUS** | 0.869 | 0.727 |
| **All except AUS plus sire in AUS** | 0.875 | 0.750 |
| **All except AUS plus sire and maternal-grand-sire in AUS** | 0.877 | 0.757 |

## *Discussion*

This paper indicates the limits to reliabilities of bulls with only foreign daughters and grand-daughters. It offers a simple check for other methods of computing the reliabilities from approximations to the inverse of the coefficient matrix.

In this paper, there is an implicit assumption that the apparent genetic correlation between proofs in different countries indicates genetic differences in true adaptation to the particular country, and is not due to differences in models or to favoured treatment of daughters of particular bulls in some countries. If this assumption is incorrect, it might be argued that a proof with a reliability of 0.85 based on daughters in several foreign countries is a more accurate indicator of merit than a proof with similar reliability based only on Australian daughters.

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