Prediction of 305-Day Milk Yield from a Limited Number of Test Days Using a Test Day Model

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Abstract

Four test day models were compared for their ability to predict 305-day milk yield production in an experimental farm with weekly milk recording. Individual lactation deviations from the average lactation curve were modeled by a constant (model LEVEL), a straight line (model LINE), a constant before the production peak and a line thereafter (model SPLINE), and Wilminks (1987b) lactation curve (model WILMINK). The LEVEL model had higher residual variance, lower correlations with true average daily milk yields and generally higher mean square error of predicted missing observations than the other models, which yielded very similar results for these statistics. The model LINE was preferred over the models SPLINE and WILMINK, because of its simplicity.

Introduction

Presently 305-day milk yields are calculated using various regression methods (see e.g. Wilmink, 1987a). A problem with these methods is that they do not account for the effect of an individual test day on the test day yield of a cow (Reents and Dopp, 1996). Further the 305-day prediction methods do not account for the individual lactation curves of a cow (Schaeffer and Dekkers, 1994), which may differ mainly due to differences in persistency. The latter effect is expected to cause the RIP-dip effect (records in progress dip) of proven bulls with high persistency and many lactations of second crop daughters in progress.

A test day model can account for the test day effect on individual test day records and for the persistency of a cow. Currently, EBV's are predicted by first predicting 305-day milk yields from the test day records and then predicting EBV's from the (predicted) 305-day yields, whereas a test day model predicts directly EBV's from the test day records. The latter implies that all effects are simultaneously accounted for. With a test day model, it is more natural to predict average daily milk yields of a cow, which is equivalent to 305-day yields, because it equals 305-day milk yield divided by 305.

A test day model should account for the stage of lactation of the cow. Many lactation functions have been described in the literature (Wood, 1968; Schaeffer et al., 1977; Batra et al., 1987; Grossman and Koops, 1988; Elston et al., 1989; Stanton et al., 1992; Scherchard et al., 1995; Gengler, 1996). Here we will fit the average lactation curve by simply fitting a class of days in milk effect. This is more flexible than any of the standard lactation curves, but it's fit costs more degrees of freedom. However, there are many thousands of test day records such that some loss of degrees of freedom hardly reduces the accuracy of the predictions.

The deviations of the individual cows from the average lactation curve at different stages of lactation, may be described by a more simple curve than a standard lactation curve. This is especially the case when we are only interested in simple statistics such as 305-day milk yield or persistency of the cows. In fact complicated curves, require estimation of many parameters, which may be a problem when the number of test days per cow is small. Loss of degrees of freedom due to many parameters for each cow can result in inaccurate estimation, because there is only a limited number of test day records per cow. Such inaccurate estimates will hamper accurate prediction of 305-day yields.

The aim of this paper is to predict average daily yields from a limited number of test days. Alternative test day models will be compared for their predictive ability for several patterns of missing test day records, such as when only a part of the lactation is known or when the interval between the test day records is varied. The aim is not to improve the current predictions of 305-day milk yields, but to develop a test day model based method to predict 305-day yields. Such a test day model would be a good base for developing breeding value estimation models for test day productions.

Data and models

The present study is based upon 951 lactations records with weekly measured test day yields from the experimental farm 't Gen (ID-DLO, The Netherlands). Milk production was measured weekly until cows were at least 150 days in milk. All lactations started between June, 1987 and April, 1996.

The test day records were analyzed by the following models:

LEVEL: $y_{ij} = \mu + ys + age + kDIM + TD + a_i + e_{ij}$

LINE: $y_{ij} = \mu + ys + age + kDIM + TD + a_i + b_i * DIM + e_{ij}$

SPLINE: $y_{ij} = \mu + ys + age + kDIM + TD + a_i + b_i * DIM^* + e_{ij}$

WILMINK: $y_{ij} = \mu + ys + age + kDIM + TD + a_i + b_i * DIM + c_i * \exp(-0.05*DIM) + e_{ij}$

y_{ij}: j^{th} test day milk yield in i^{th} lactation (lactations are assumed uncorrelated)
μ: intercept
ys: year-season of calving (3 monthly classes per year)
age: age at calving (4 monthly classes)
kDIM: class of days in milk * parity (weekly classes with parity defined as first and later)
DIM: number of days in milk at the test day
DIM^*: $DIM^* = DIM - 42$ for $DIM \geq 42$; otherwise $DIM^* = 0$
a_i: random effect of the lactation (genetic plus within lactation permanent environment effect)
b_i: random regression coefficient on days in milk for i^{th} lactation
c_i: random regression coefficient on the Wilmink (1987b) factor for i^{th} lactation
e_{ij}: residual

In all four models the average lactation curve is described by the effect of the class of days in milk (weekly classes within first and later parity; kDIM). The a_i, b_i and c_i terms, model the deviation of the individual lactation curve from the average lactation curve expressed by kDIM. The model LEVEL includes only the term a_i, which implies that the deviation from the average lactation curve is assumed to be constant over the whole lactation for each individual lactation. In the model LINE, the deviation from the average lactation curve is expected to be a straight line for each individual lactation which is modelled by the terms a_i and b_i. The b_i term expresses the individual deviation of the average slope of the average lactation curve and may be interpreted as the persistency of the lactation. In the SPLINE model, the deviation is assumed constant for the first 42 days of lactation (until the peak production), and is assumed to follow a straight line after the peak production, with the slope b_i representing persistency. In the WILMINK model, the deviation from the average lactation curve is expected to follow the Wilmink (1987b) curve: $a_i + b_i * DIM + c_i * \exp(-0.05*DIM)$. The random regression on exp(-0.05*DIM) for each individual lactation, could model a fast change at the begin of the lactation curve followed by an almost straight line.

REML estimation of variances

Variances and covariances were estimated by the EM-REML algorithm:

$$\sigma_{\alpha\beta} = (\hat{\alpha}_i{}' \hat{\beta}_i + tr(C_{\alpha\beta})\, \sigma_e^2)/q \quad \text{and}$$

$$\sigma_e^2 = y'\, \hat{e}/(n-rank(x))$$

where

$\hat{\alpha}_i(\hat{\beta}_i)$: vector with MME solutions for term $\hat{\alpha}_i(\hat{\beta}_i)$, with $\alpha_i(\beta_i)$ being a_i, b_i or c_i

$C_{\alpha\beta}$: part of the inverse MME which corresponds to the equations for α and β

q: number of effects in α (equals number of effects in β)

y: data vector

ê: vector with estimated residual terms

n: number of observations

rank(x): rank of the fixed effect part of the MME

Comparison of models

The goodness of fit of modelling individual deviations from the average lactation curve, for the different models, is investigated by analyzing the complete data set, (i.e. all known weekly test days records are included), and several subsets, (i.e. part lactations and varied lengths of test day intervals). Differences in goodness of fit between the models are expressed by:

1) the residual variances

2) correlations between predicted average daily milk yield ($\bar{y}_i$ = 305 day yield / 305):

LEVEL: $\bar{y}_i = a_i$

LINE: $\bar{y}_i = a_i + b_i * (305/2)$

SPLINE: $\bar{y}_i = \{42a_i + 263 * (a_i + 131 * b_i)\}/305$

WILMINK: $\bar{y}_i = a_i + b_i * (305/2) + c_i * \int_1^{305} \exp(-0.05*DIM)\, dDIM / 305$

the $\bar{y}_i$ of the sub data sets were correlated to those off the complete data set.

3) the mean square error of predictions of missing observations:

MSEP = 3(y_i-$č_i$)5/n

where

y_i : test day record that is missing in the sub data set, but known in the complete data set

$č_i$: predicted value of missing record using the models LEVEL, LINE, SPLINE and WILMINK

n: number of missing records in the sub data set

Results

Residual variances

REML estimates of the (co)variances of the terms a_i, b_i and c_i are in Table 1. Including the b_i term in the model LINE reduces the residual variance with 2.8 kg5 daily milk yield. The residual variance for the models LINE, SPLINE and WILMINK are almost identical. The variance of the a_i and b_i terms are equal for the models LINE and WILMINK which indicates that the regression on the exp(-0.05*DIM) term hardly affects the predictions. According to the residual variances the model SPLINE is slightly better than LINE and WILMINK, which are better than LEVEL.

Table 1. REML estimates of variance and covariance for the a_i, b_i and c_i terms from the LEVEL, LINE, SPLINE and WILMINK models (in kg milk5).

Covariance	LEVEL	LINE	SPLINE	WILMINK
a_i,a_i	17.97047	21.66397	19.61924	21.66887
a_i,b_i		-0.04648	-0.33038	-0.04651
b_i,b_i		0.00055	0.00064	0.00055
a_i,c_i				-0.00406
b_i,c_i				0.00000
c_i,c_i				0.00005
e_i,e_i	9.39848	6.56309	6.555015	6.56300

Correlations between predicted average daily milk yields

The correlation between predicted average daily milk yield in the sub sets for all four models is compared with the predicted average daily milk yield in the completed data set for the model itself and for model LINE. The latter because it was assumed that INE (or WILMINK) yielded the best prediction of average daily yield in the complete data set. For part-lactations (Table 2) correlations improve with the number of days in milk and are highest for LINE and WILMINK, lower for SPLINE and the lowest for LEVEL. The WILMINK and LINE correlations are equal because of the negligible effect of the c_i term.

When the intervals between test days were varied (Table 3) correlations decreases with the length of the interval, but differences are relative small. As before, the correlations for LINE and WILMINK are equal.

Table 2. Correlations between predicted average daily milk yield in part lactation sub sets with the complete data set from the LEVEL, LINE, SPLINE and WILMINK models.

		Correlations of sub sets with complete data set (indicated by *)					
DIM in sub set[1)]	Number of records	LEVEL LEVEL*	LEVEL LINE*	LINE LINE*	SPLINE SPLINE*	SPLINE LINE*	WILMINK LINE*
complete	36,288	1.000	.9748	1.000	1.000	.9986	1.000
# 147	27,978	.9480	.9067	.9285	.9192	.9226	.9285
# 133	27,028	.9372	.8920	.9105	.8997	.9038	.9105
# 119	26,068	.9240	.8742	.8940	.8833	.8887	.8940
# 105	25,118	.9063	.8504	.8786	.8735	.8806	.8786
# 91	24,171	.8829	.8216	.8597	.8634	.8719	.8597
# 77	23,222	.8530	.7869	.8335	.8367	.8467	.8335
# 63	22,144	.8199	.7498	.8016	.7821	.7925	.8016
# 49	21,223	.7746	.7025	.7515	.7049	.7136	.7515
# 35	20,281	.7040	.6292	.6557	.6240	.6312	.6557
# 21	19,352	.6194	.5444	.5530	.5411	.5469	.5530

[1)] Part-lactation are created by taking a specified period of the complete lactation. This is done for only 50 per cent of the animals (randomly chosen) so that all other fixed effects can be estimated.

Table 3. Correlations between predicted average daily milk yield in sub sets with varied test day intervals with the complete data set from the LEVEL, LINE, SPLINE and WILMINK models.

		Correlation of sub sets with complete data set (indicted by *)					
Milkrec. freq.[1)]	Number of records	LEVEL LEVEL*	LEVEL LINE*	LINE LINE*	SPLINE SPLINE*	SPLINE LINE*	WILMINK LINE*
Weekly[2)]	36,288	1.000	.9748	1.000	1.000	.9986	1.000
2-weekly	18,120	.9970	.9724	.9957	.9930	.9951	.9957
3-weekly	9,032	.9943	.9678	.9907	.9845	.9903	.9907
4-weekly	7,231	.9881	.9637	.9833	.9759	.9828	.9833
5-weekly	6,057	.9828	.9557	.9766	.9643	.9767	.9766
6-weekly	5,180	.9837	.9566	.9770	.9631	.9773	.9770
7-weekly	4,520	.9753	.9515	.9695	.9552	.9700	.9695
8-weekly	4,042	.9673	.9441	.9619	.9497	.9619	.9619
9-weekly	3,566	.9557	.9280	.9513	.9318	.9510	.9513
10-weekly	3,293	.9585	.9351	.9522	.9314	.9520	.9522

[1)] sub sets are generated by deleting all records for a specific test day for all animals.
[2)] complete data set, weekly intervals.

Mean square error of predictions of missing observations

In order to compare the different models LEVEL, LINE, SPLINE and WILMINK for their predictive ability, mean square errors of prediction of missing observations (MSEP) for different lengths of part lactations are presented in Table 4 and for varied test day intervals in Table 5. Results show that MSEP is less sensitive for longer between test intervals than for part lactations. The goodness of fit increases when the intervals become shorter and when the length of part lactations becomes longer. This is clear for all four models for varied intervals but less clear for LINE, SPLINE and WILMINK when the length of the part lactations increases. Instead of decreasing, the MSEP increases as the length of the part lactation increases from 77 to 133 days, which may be due to a too high weight of the information of the test days for the prediction of the slope of the line, which then deviates too much from zero. The too high weight may be due to the imperfect model, which becomes mainly apparent in the extrapolation that is needed to predict $\check{c}_i$.

Table 4. Mean square error of predictions of missing observations in part lactation sub sets from the LEVEL, LINE, SPLINE and WILMINK models.

DIM in sub set	Number of records	Number of pred.rec.	Mean square error of pred. of missing rec. LEVEL	LINE	SPLINE	WILMINK
Complete	36,288					
# 147	27,978	8,310	15.78	21.61	22.41	21.61
# 133	27,028	9,260	15.83	22.64	23.14	22.64
# 119	26,068	10,210	16.01	23.15	22.96	23.15
# 105	25,118	11,160	16.51	22.16	20.52	22.16
# 91	24,171	12,107	17.30	20.71	17.48	20.71
# 77	23,222	13,056	18.27	20.03	16.75	20.03
# 63	22,144	14,004	19.21	19.60	17.97	19.60
# 49	21,223	14,921	20.57	19.90	20.05	19.90
# 35	20,281	15,862	22.39	21.29	21.85	21.29
# 21	19,352	16,657	24.48	23.32	23.90	23.32

Table 5. Mean square error of predictions of missing observations in sub set with varied test days intervals from the LEVEL, LINE and WILMINK models.

Milk recording frequency[1)]	Number of records	Number of pred.rec.	Mean square error of pred. of missing rec. LEVEL	LINE	SPLINE	WILMINK
Weekly[2)]	36,288					
2-weekly	18,120	8,946	9.64	6.97	6.91	6.97
3-weekly	9,032	11,885	10.17	7.54	7.49	7.54
4-weekly	7,231	13,431	10.07	7.59	7.55	7.59
5-weekly	6,057	14,342	10.38	7.98	7.93	7.98
6-weekly	5,180	14,902	10.40	8.17	8.11	8.17
7-weekly	4,520	15,309	10.70	8.59	8.54	8.59
8-weekly	4,042	15,655	10.81	8.72	8.70	8.72
9-weekly	3,566	15,876	10.96	8.97	8.96	8.97
10-weekly	3,293	16,129	11.15	9.29	9.30	9.29

1) Sub sets are generated by deleting all records from a test days for 50 per cent of the animals (randomly chosen) so that all other fixed effects needed to predict the missing records can be estimated.
2) complete data set

Discussion and conclusions

Four test day models LEVEL, LINE, SPLINE and WILMINK were compared for the goodness of fit for average daily milk yield based on weekly milk yield data, recorded in an experimental herd. Differences between the models in goodness of fit were small. The model LINE, SPLINE and WILMINK had substantially lower residual variances, which suggests that these models should be preferred over LEVEL. The model LINE gave identical results as the model WILMINK, which implies that model LINE should be preferred because of simplicity. The LINE model had higher correlations with "true" average daily milk production, estimated in the complete data set. The only shortcoming of LINE is a improper extrapolation of missing observations. This is probably because polynomials, such as LINE, yield poor predictions of extrapolated records. The poor prediction of extrapolated records may result in larger changes of $\bar{y}_i$ as the lactation progresses. This was not seen in the correlations (Table 2), but the variances of the predicted $\bar{y}_i$ were higher with LINE than with LEVEL (unpublished results).

In conclusion the model LINE combines simplicity with a high level of goodness of fit of average daily milk yields. It seems therefore to provide a simple and good model from which test day models for breeding value estimation can be developed.

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