Approximations to Interbull Reliabilities

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Jones (1997) showed that the reliabilities calculated by Interbull for some bulls were higher than is possible from the correlations between performance in different countries. The bulls where this seems to have occurred were those with many sons.

For a single trait sire model, Robinson & Jones (1987) found that most approximations tended to underestimate the diagonal of the inverse (i.e. overestimate the reliability) if sires were related. They showed that for several approximations, a better approximation could be obtained if the approximation was obtained by

firstly assuming that sires are unrelated,
secondly adding information for relatives.

Thus if a bull has n effective daughters, and its sire has m effective daughters the reliability of the bull's breeding value is approximated by (3mn + 4kn + km)/ (3mn+4km+ 4kn+ $4k^2$) where k is the increment to the diagonal of the coefficient matrix {k $=(1-h^2/4)/(h^2/4)$}. This is equivalent to adding mk/(3m+4k) to the effective daughters of the son. Similarly nk/(3n+4k) is added to the effective daughters of the sire. The maximum contribution from sire to son is k/3 effective daughters. Since a sire can have many sons, its sons can make a substantial contribution to its reliability. In a similar way add nk/(15n+16k) to the effective daughters of the maternal-grandsire.

Wilmink and Dommerholt (1985) also found that adding relationships separately gave a better estimate of reliability.

Of the approximations considered by Robinson & Jones (1987), simply using the reciprocal of the diagonal overestimated the reliability most. This approximation is equivalent to the one used by Interbull.

Robinson & Jones suggested that a better approximation for the multiple trait model would be

$$C^{ii} = \left\{ C_{ii} - \sum_{i \neq j}^{n} C_{ij} C_{ij}^{-1} C_{ji} \right\}^{-1}$$

for i=1,2 ....,n**.**

where C is the coefficient matrix, C^{-1} the inverse and C_{ij} , C^{ij} are partitioned matrices of C or C^{-1}. This takes better account of the off-diagonal terms.

A similar approximation was given by Wang et al. (1995)

$$C^{ii} = C_{ii}^{-1} + C_{ii}^{-1} \sum_{i \neq j}^{n} \left[C_{ij} C_{jj}^{-1} C_{ji} \right] C_{ii}^{-1}$$

for i=1,2 ....,n.

The degree to which reliabilities are overestimated can be seen from an example given by Robinson and Jones for a single trait model. For a case with a sire and two sons but none having daughters k$\mathbf{A}^{-1}$ **=**

$$\begin{bmatrix} 5k/3 & -2k/3 & -2k/3 \\ -2k/3 & 4k/3 & 0 \\ -2k/3 & 0 & 4k/3 \end{bmatrix}$$

and the diagonals of the inverse should be 1/k.

The approximation of Robinson and Jones gives 1/k,15/16k and 15/16k. This gets the correct term for the sire but for k=15 adds one effective daughter to the sons' reliability. In the limit, the sons' reliabilities are calculated as if the sire had a very high reliability.

The approximation of Wang gives 21/25k, 18/20k,18/20k. This is equivalent to adding 2.8 effective daughters to the sire and 1.7 to each son. In the limit, it adds 2.5 effective daughters per son to the sire, and the sons' reliabilities are calculated as if the sire had a very high reliability.

The Interbull method would give approximations of 3/5k, 3/4k, 3/4k for the diagonals of the inverse.

Effectively this adds 5 effective daughters per son to the sire, and the sons' reliabilities are calculated as if the sire had a very high reliability.

The above example illustrates why the reliabilities of some bulls were overestimated by Interbull. Where a sire has many sons the approximation assumes that the sons have a high reliability, even though they have no daughters in a particular country. The approximation of Robinson and Jones would give a better estimate of the reliability of the sire, but would still overestimate the reliability of each son.

Robinson and Jones (1987) showed that it is generally better to consider relationships as a separate stage adding the appropriate number of effective daughters.

Suggested approach

Step 1
For each country, list the number of effective daughters for each bull, ignoring pedigree.

Step 2
Adjust the effective daughter number of each sire by adding nk/(3n+4k) effective daughters for each son, starting from the youngest bulls. In a similar way, add nk/(15n+16k) effective daughters to the maternal-grandsire.

Step 3
In turn for each bull, use the method of Jones (1997) to compute the reliability for performance in each country. The reliabilities from step 3 are used in setting up the ***P*** and ***G*** matrices to compute

$$b = P^{-1}G$$

where ***P*** gives the correlation among the proofs available, and ***G*** gives the correlation between the foreign proof and the true genetic merit in the target country.

$$P = \begin{bmatrix} R_1^2 & R_1^2 R_2^2 r_{12} \\ R_1^2 R_2^2 r_{12} & R_2^2 \end{bmatrix} \quad G = \begin{bmatrix} R_1^2 r_{1A} \\ R_2^2 r_{2A} \end{bmatrix}$$

The reliability of the breeding value in the target country (A) is given by

$$R_A^2 = r_{AI}^2 = b'Pb$$

Step 4
Adjust the effective daughter number of each son by adding mk/(3m+4k) effective daughters for its sire, starting from the oldest animals. The appropriate value for m is the effective daughter number for the sire in the absence of the contribution from this son. In the same way, adjust for the contribution of the maternal-grandsire by adding mk/(15m+16k).

Example

Assume bull A has 3 sons (B,C,D) with the following effective daughter numbers, ignoring pedigrees of bulls.

A USA 1000
B USA 30
C NZL 30
D AUS 30

Assume the heritability is 0.25 in Australia (k=15) and 0.30 in USA and NZL (k=12.333), and the apparent genetic correalations between performance in USA and NZL is 0.77, between performance in USA and AUS is 0.81, and between performance in AUS and NZL is 0.90.

Step 1
The number of effective daughters supplied for each country is

	Effective daughters in each country		
Bull	USA	NZL	AUS
A	1000	0	0
B	30	0	0
C	0	30	0
D	0	0	30

Step 2
Add nk/(3n+4k) to the effective number of A in USA to account for the contribution from B. Make similar adjustments to A for contribution of C in NZL and D in AUS. Convert effective daughters to reliabilities.

Bull	Effective daughter number			Reliability		
	USA	NZL	AUS	USA	NZL	AUS
A	1003	2.65	3	0.988	0.177	0.167
B	30			0.709		
C		30			0.709	
D			30			0.667

Step 3
The matrices for A would be

$$P_A = \begin{bmatrix} .988 & .135 & .133 \\ .135 & .177 & .027 \\ .133 & .027 & .167 \end{bmatrix}$$

$$G_{A.USA} = \begin{bmatrix} .988 \\ .136 \\ .135 \end{bmatrix} \quad G_{A.NZL} = \begin{bmatrix} .761 \\ .177 \\ .150 \end{bmatrix} \quad G_{A.AUS} \begin{bmatrix} .800 \\ .159 \\ .167 \end{bmatrix}$$

The reliability of A in USA, NZL and AUS is 98.8%, 63.2% and 68.5%.

The reliabilities and effective daughters are now

Bull	Reliability			Effective daughter number		
	USA	NZL	AUS	USA	NZL	AUS
A	0.988	0.632	0.685	1003	21.2	32.6
B	0.709	0.420	0.465	30	8.9	13.0
C	0.420	0.709	0.574	8.9	30	20.2
D	0.438	0.540	0.667	9.6	14.5	30

Step 4
Adjust the effective daughters of B, C and D for the contribution from A. In this case, the appropriate number of effective daughters from A to use is that which would have been found if the effect of the son had been ignored. For B, these would have been 1000 in USA, 21.2 in NZL and 32.6 in AUS.

Similarly for C, these would have been 1003 in USA, 17.4 in NZL and 32.6 in AUS, and for D 1003 in USA, 21.2 in NZL and 17.6 in AUS.

The effective daughters and reliabilities are now

Bull	Effective daughter number			Reliability		
	USA	NZL	AUS	USA	NZL	AUS
A	1003	21.2	32.6	0.988	0.632	0.685
B	34.0	11.2	16.1	0.734	0.476	0.518
C	12.9	32.3	23.3	.511	0.724	0.608
D	13.6	16.8	32.3	0.524	0.576	0.683

For most bulls, the effect of ignoring the contribution of a son to the sire, when allowing for that of the sire will be small.

The reason for the higher reliability of A in AUS than in NZL is the higher genetic correlation between USA and AUS (0.81) than between USA and NZL (0.77).

This method is very similar to that presented by Harris and Johnson (1998) at this conference. Johnson's method is neater in that calculations are all done in reliability. This method has the advantage that the effects of sons are simply added.

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